Internal shock waves in relativistic jets

SERGIO MENDOZA <sergio@mendozza.org>
http://www.mendozza.org/sergio
INSTITUTO DE ASTRONOMÍA
UNIVERSIDAD NACIONAL AUTÓNOMA
DE MÉXICO (UNAM)

Talk available at
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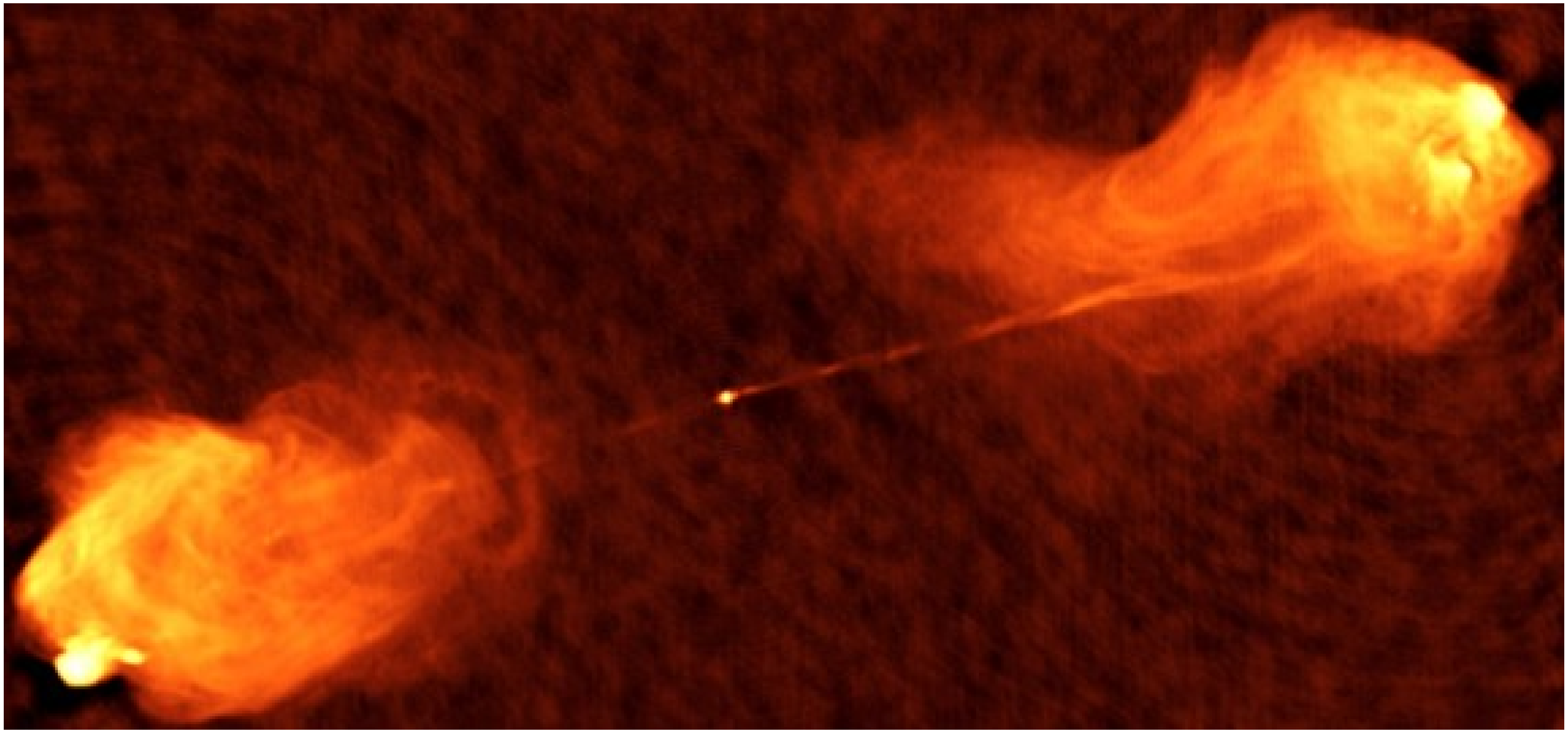
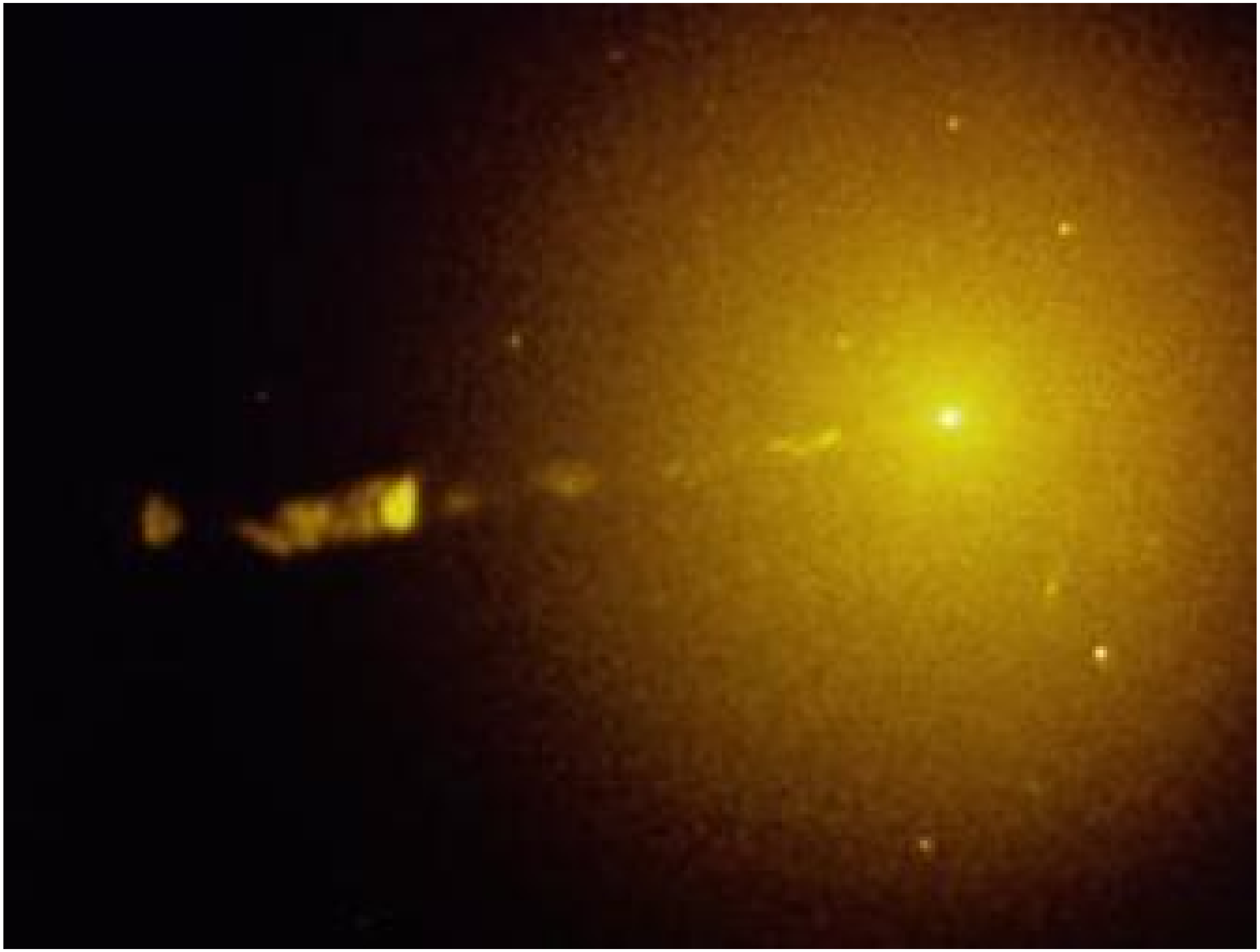
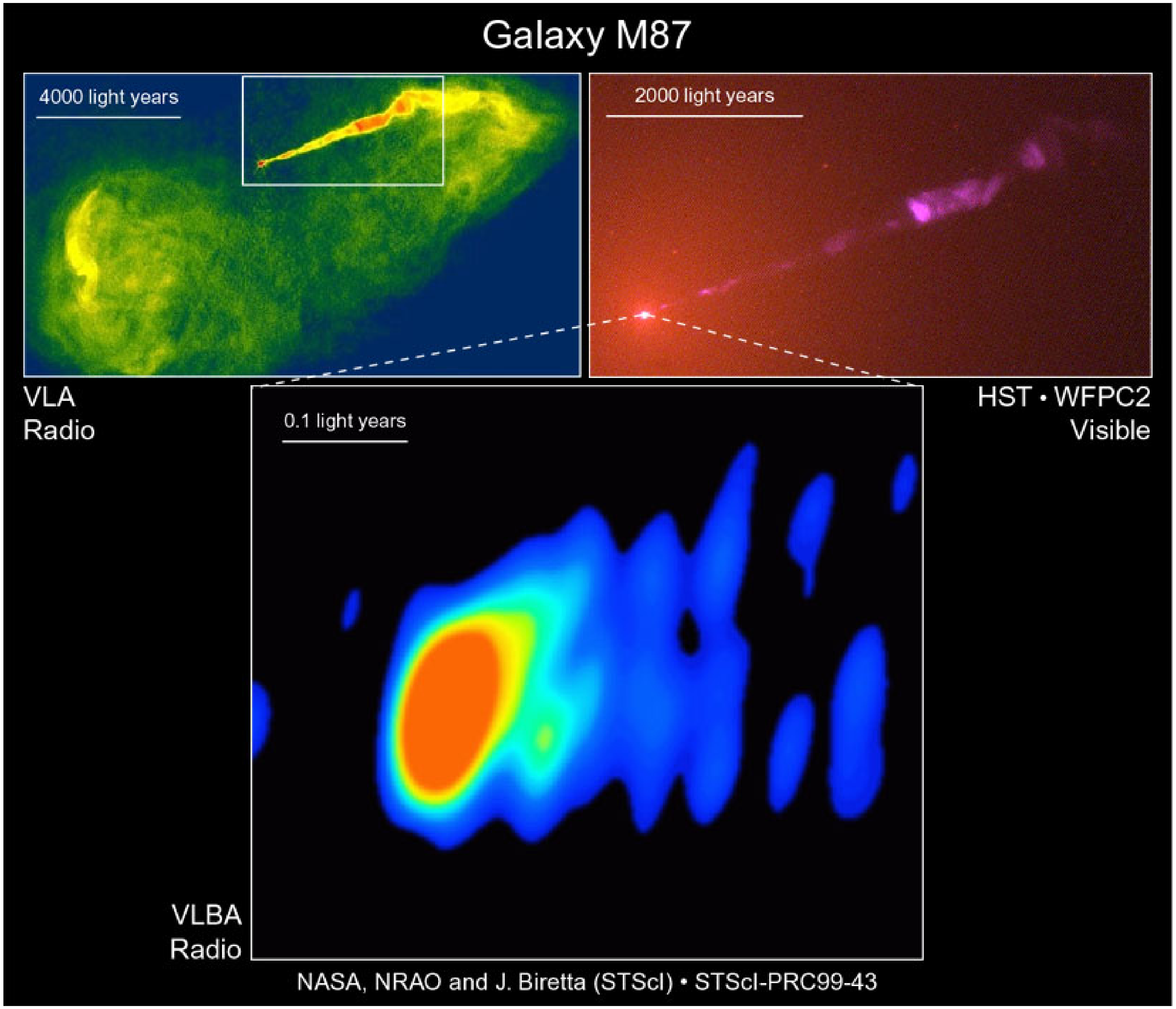
Collaborators

- ☆ Xavier Hernandez, Juan Carlos Hidalgo, Tula Bernal, Yaxkin Coronado, Luis Torres, Daniel Olvera (IA-UNAM)
- ☆ José Ignacio Cabrera (IF-UNAM)

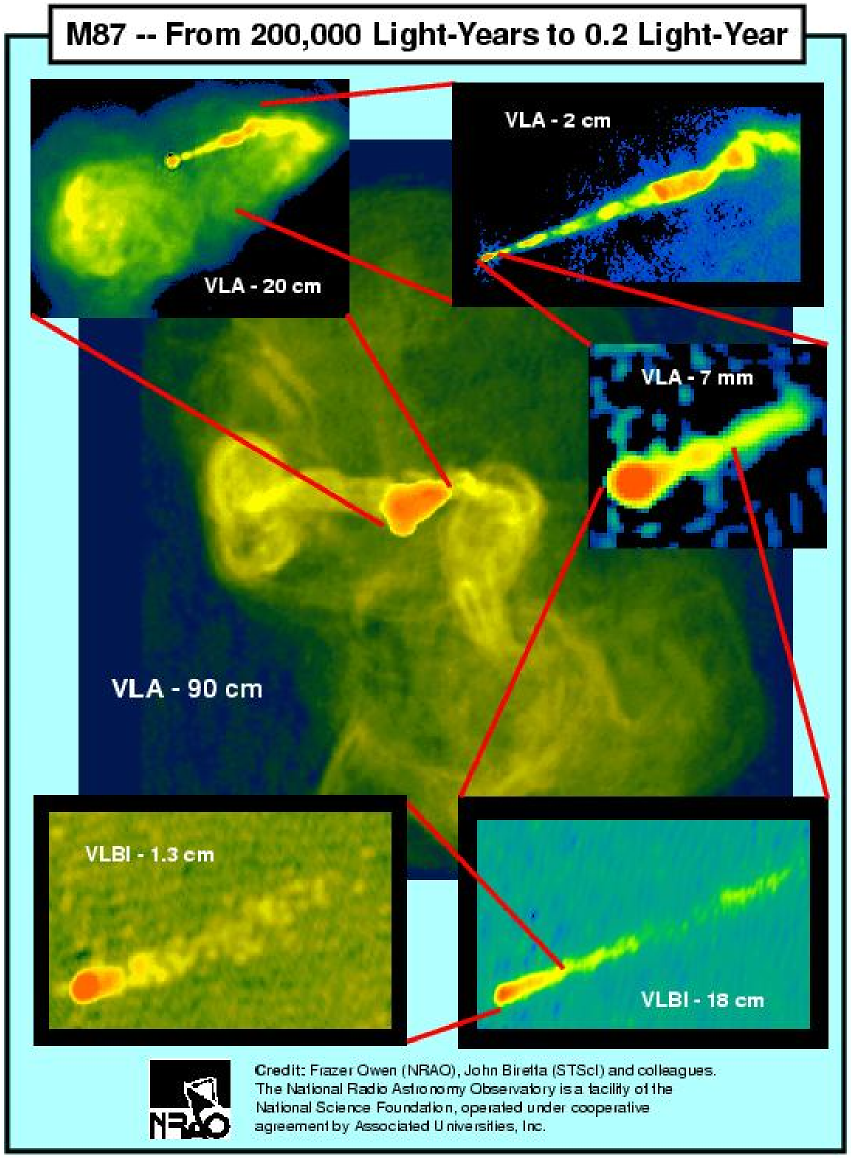




M87 as seen by the HST.



M87 at different scales and wavelengths.



M87 -- From 200,000 Light-Years to 0.2 Light-Year

Credit: Frazer Owen (NRAO), John Biretta (STScI) and colleagues.
The National Radio Astronomy Observatory is a facility of the National Science Foundation, operated under cooperative agreement by Associated Universities, Inc.

1 Relativistic hydrodynamics

- Perfect fluid. Quantities measured on the proper system of reference.
- Quantities measured per unit volume.
- Gravity is given: $g_{\mu\nu}$

$$\nabla_\mu \left(n u^\mu\right) = 0, \qquad \text{(continuity)} \tag{1}$$

$$\nabla_\mu T^{\mu\nu} = 0, \qquad \text{(energy-momentum)} \tag{2}$$

$$\nabla_\mu \left(\sigma u^\mu\right) = 0, \qquad \text{(entropy)} \tag{3}$$

$$w u^\mu \nabla_\mu u_\nu = \partial_\nu p - u_\nu u^\mu \partial_\mu p. \qquad \text{(Euler)} \tag{4}$$

- Enthalpy density: $w = e + p$, $T_{\mu\nu} = w u_\mu u_\nu + p g_{\mu\nu}$.
- Polytropic relation: $p \propto n^\kappa$

$$e = mnc^2 + \varepsilon = mnc^2 + p/\left(k+1\right) \tag{5}$$

- Extremely energetic flows (ultrarelativistic at the microscopic level): $\epsilon \ll p$ $\Rightarrow p = (\kappa + 1)$ (Bondi-Wheeler equation).

2 Shock waves

- Strong discontinuity of a flow at supersonic velocities.
- Flux conservation of particle, momentum and energy with a particle flux crossing the discontinuity.
- Taub's conditions (relativistic Hugoniot jump conditions)
- Very general arguments imply that:

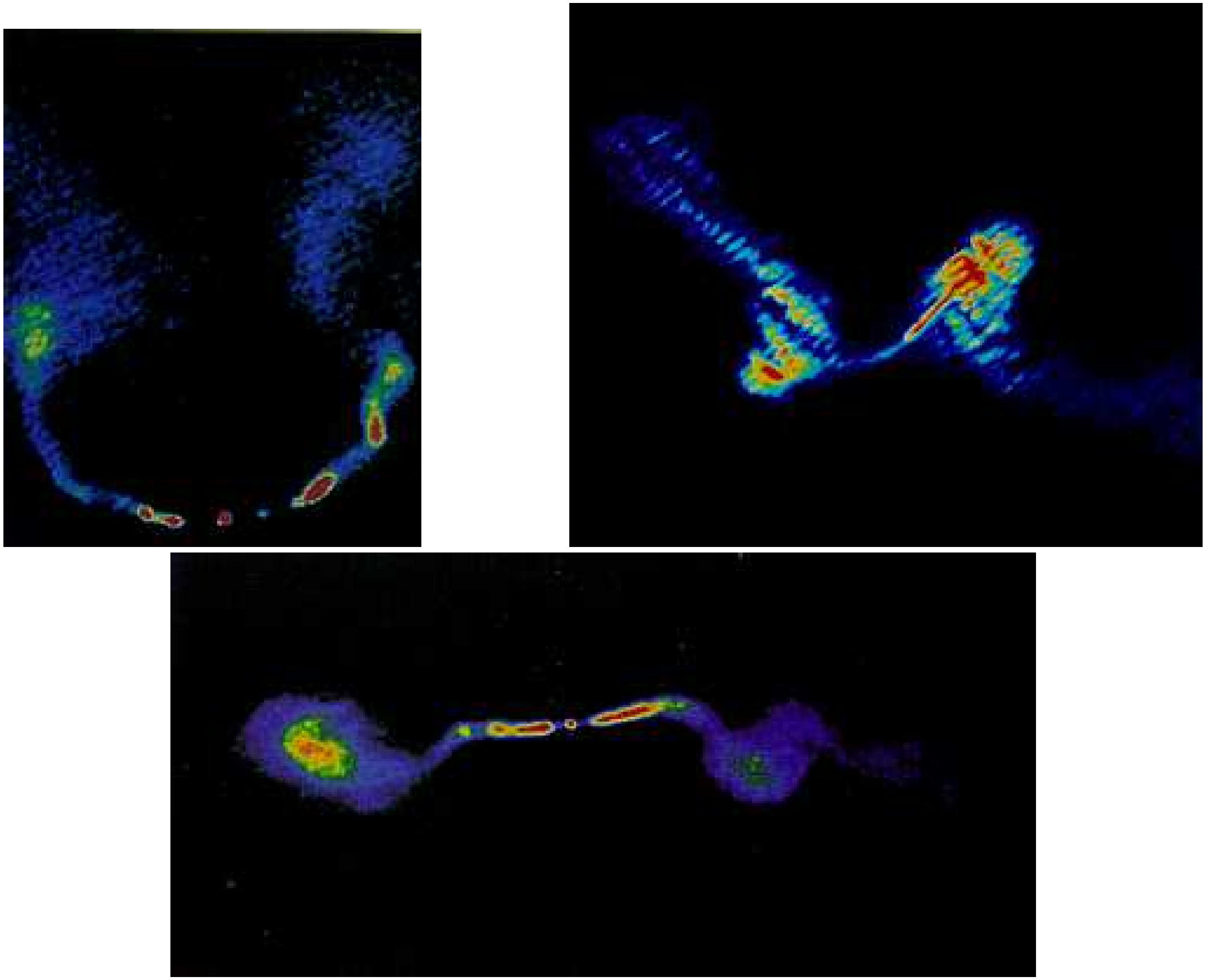
$$p_2 > p_1, \qquad n_2 > n_1, \qquad v_1 > a_1, \qquad v_2 < a_2$$

- McKee & Colgate (1973) developed jump conditions adequate to any system of reference.

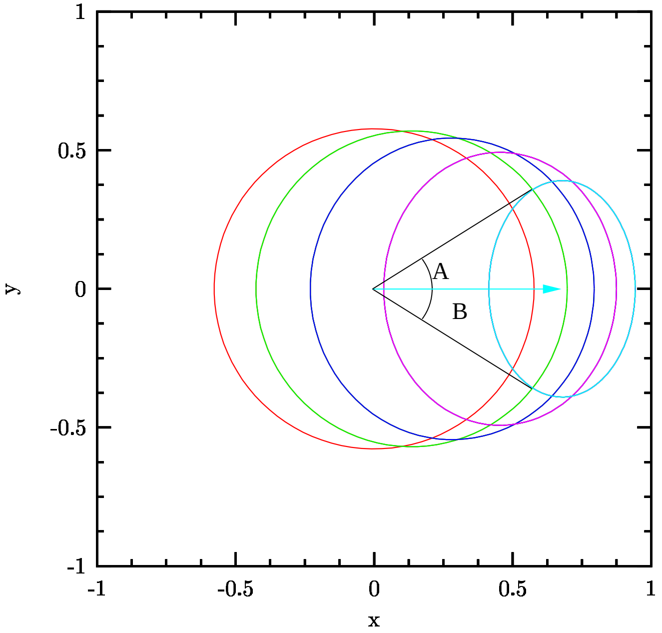
$$\frac{e}{\rho} = \frac{\gamma}{\rho_1}\left[e_1 + p_1 \frac{\beta}{\beta_{\mathrm{s}}}\right], \qquad \frac{n}{n_1} = \gamma \frac{\kappa}{\kappa - 1} + \frac{1}{\kappa - 1},$$

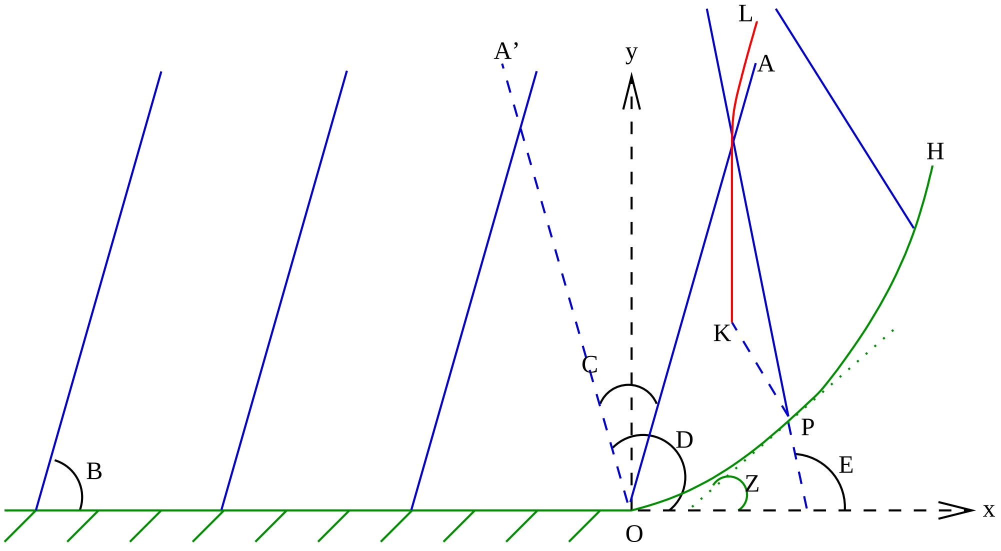
$$\Gamma = \frac{\gamma}{\sqrt{1 - 2\left(\kappa - 1\right)/\kappa}}.$$

3 Bent jets



- Characteristics. Mach number: $M = 1/\sin\alpha = \gamma v/\gamma_a a$





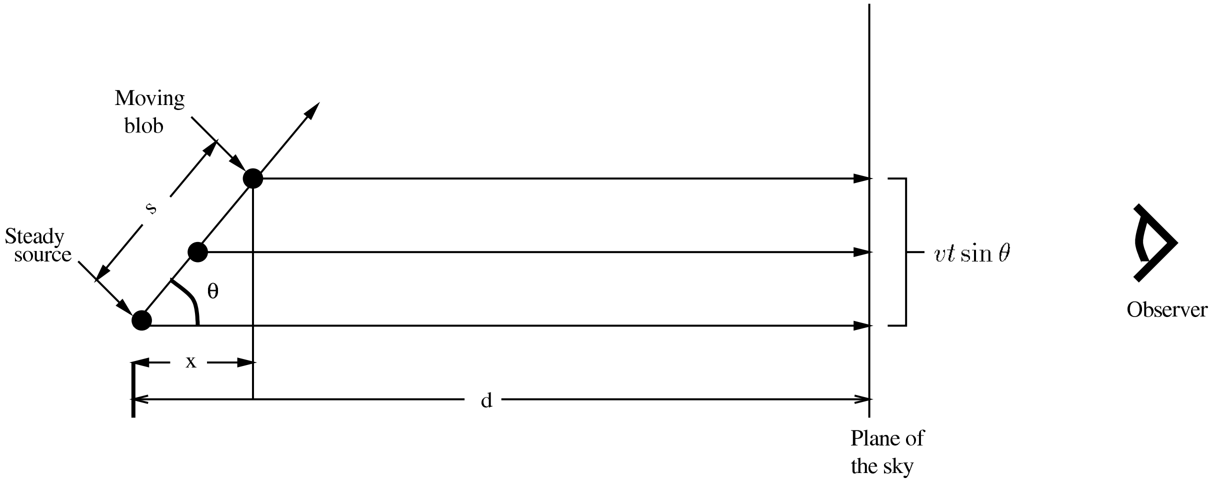
- Maximum deflection angle given by (Mendoza & Longair 2002):

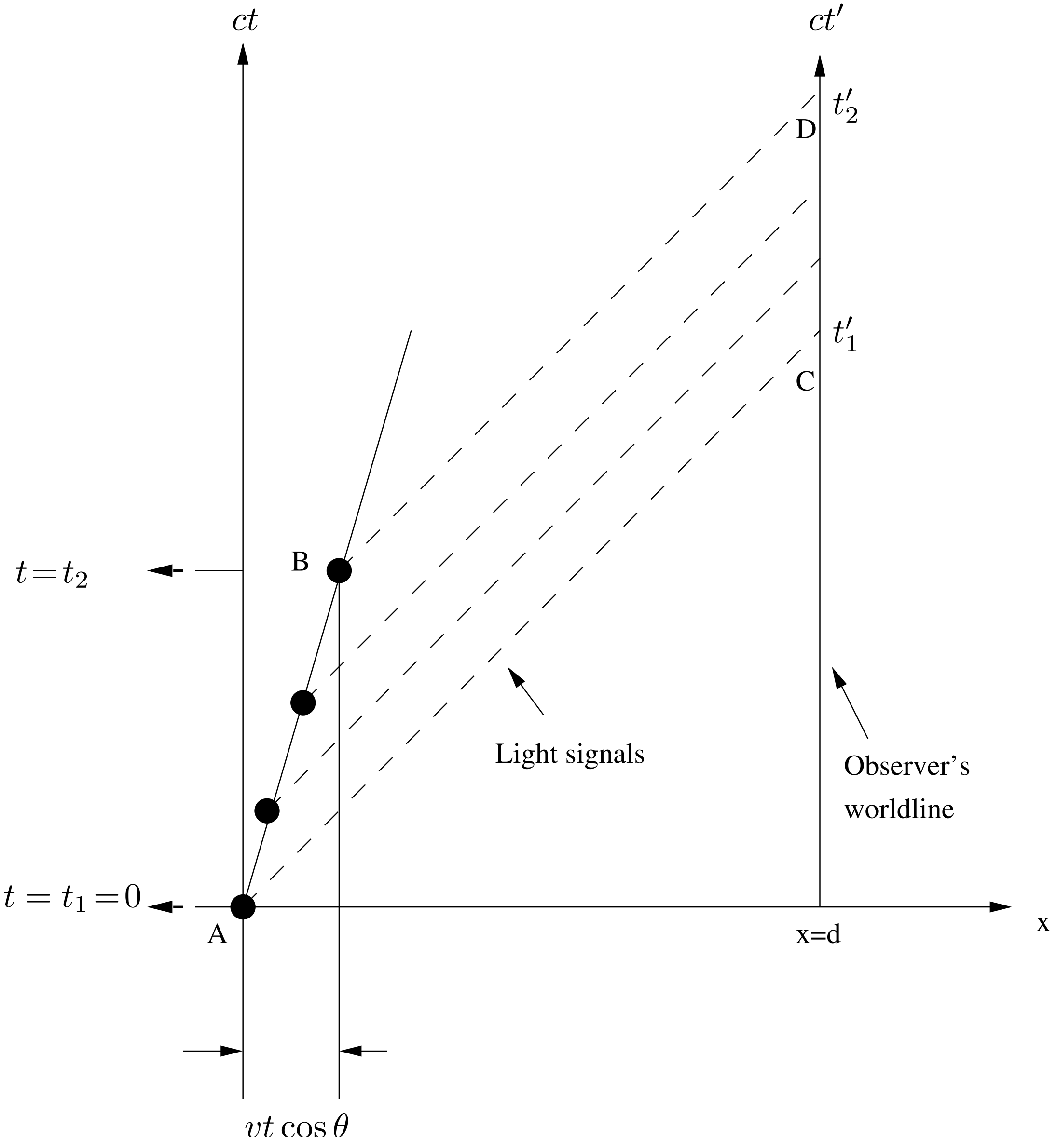
$$\theta_{\max} = \begin{cases} 74.21^\circ & \text{Relativistic, } \kappa = 5/3 \\ 47.94^\circ & \text{Relativistic, } \kappa = 4/3 \\ 134.16^\circ & \text{Newtonian, } \kappa = 4/3 \end{cases}$$

- The more relativistic a jet is, the harder it is to bend it (Mendoza & Longair 2001).

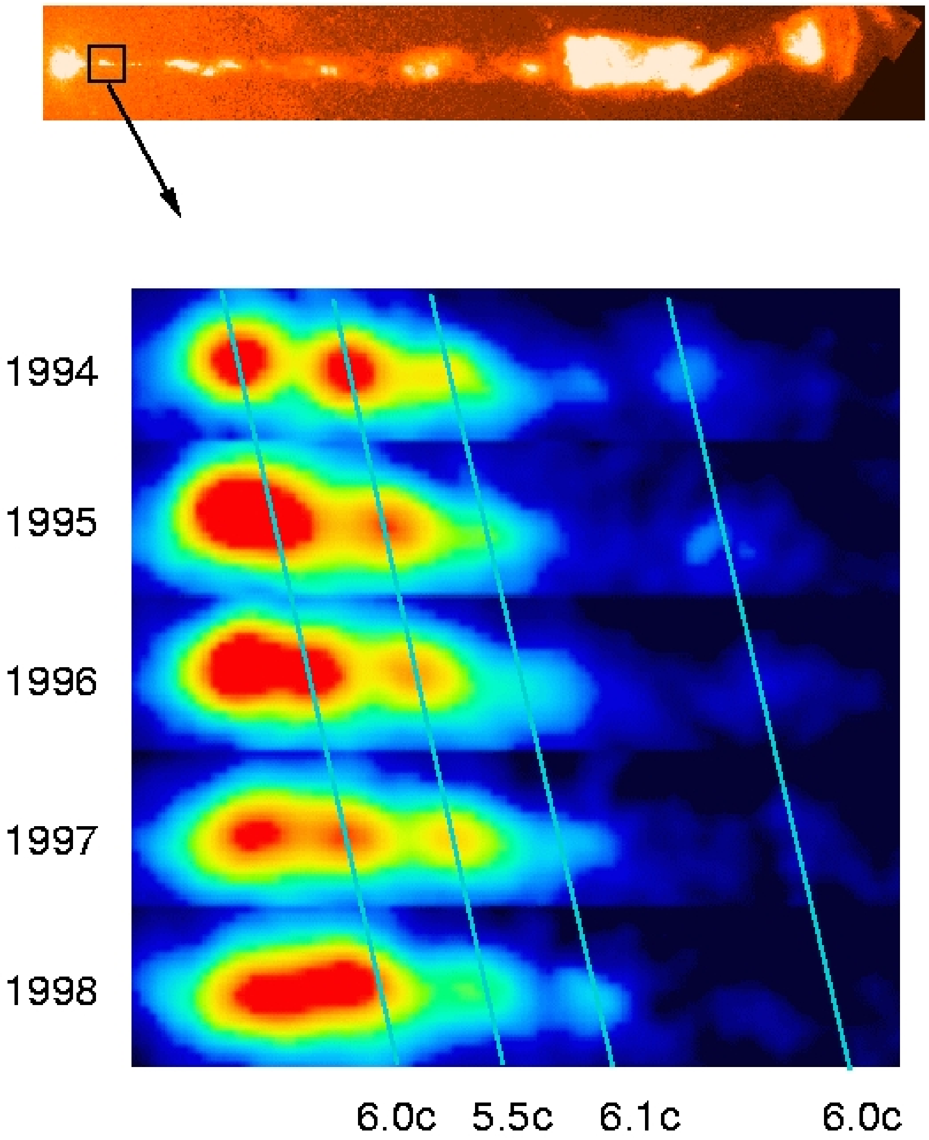
4 Apparent superluminal motion in jets

- ☆ Theoretical prediction by Rees (1966).
- ☆ There are sources in the sky which apparently move at velocities greater than light.
- ☆ Problem: astronomers measure velocity as observed distance divided by measured time.
- ☆ Solution: a projection problem on the sky due to relativistic motions very close to the speed of light ($v \gtrsim 0.99c$).



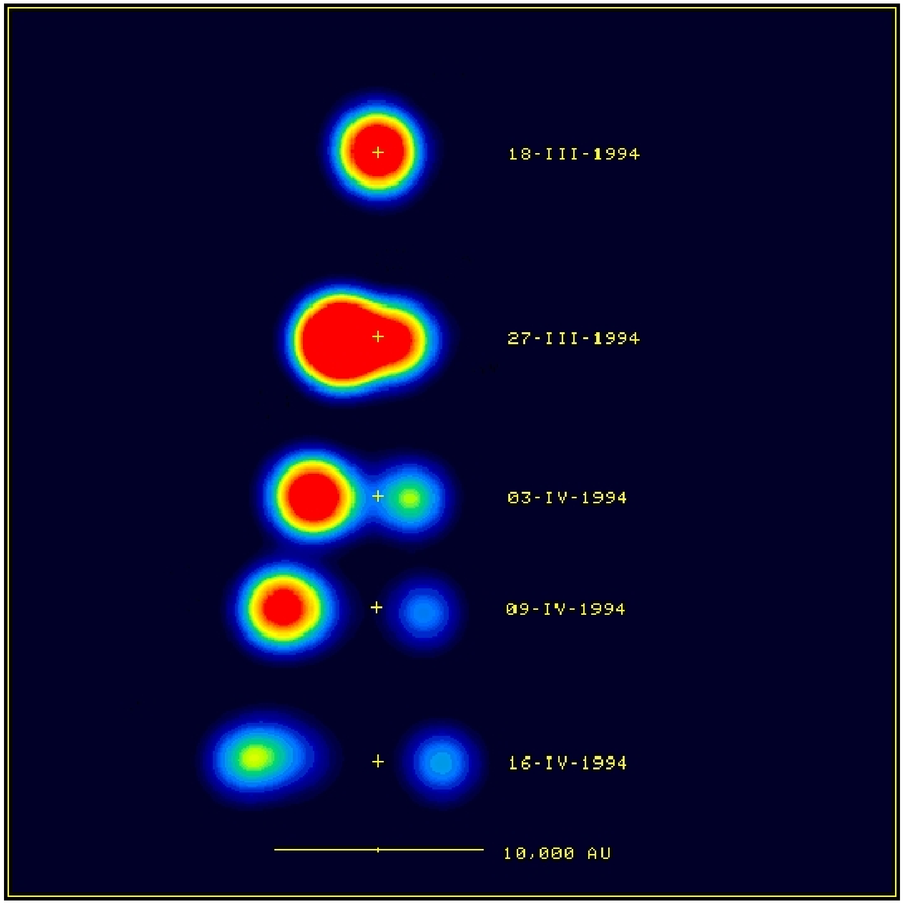


Superluminal motion on a space-time diagram (Mendoza 2003)

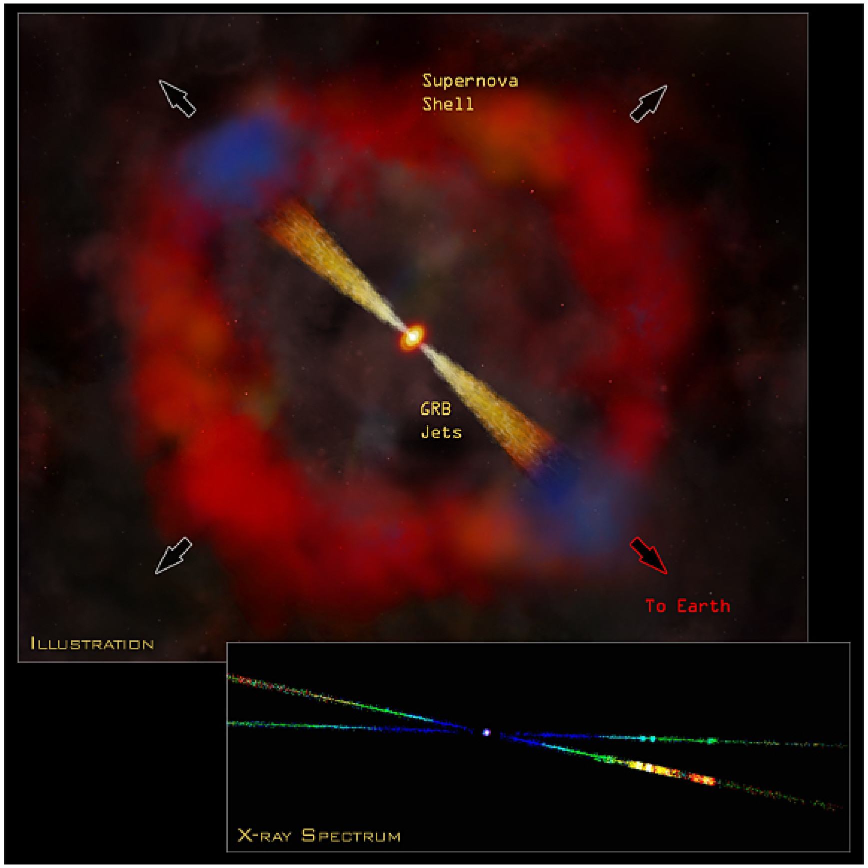


Apparent superluminal motion on M87's jet

5 Micro-qsr & long γ-ray burst jets



GRS 1915+105 (Mirabel & Rodríguez 1994)



Fireball's model cartoon (Meszaros & Rees, M.J. 1997)

6 Working surfaces

- ☆ Cantó, et. al (2000) & Cantó, et. al (2003) made non–relativistic models with ballistic approximations to working surfaces inside jets.
- ☆ Mendoza, Hidalgo et al. (2009) generalised their model to the relativistic regime as follows.

- ☆ Analysis in 1D.
- ☆ Velocity $u(\tau)$ with mass discharge $\dot{m}$.
- ☆ Ejection mechanism behaves as free streaming.
- ☆ In this case a working surface is formed due to the fact that fast flow overtakes slow fluid particles.

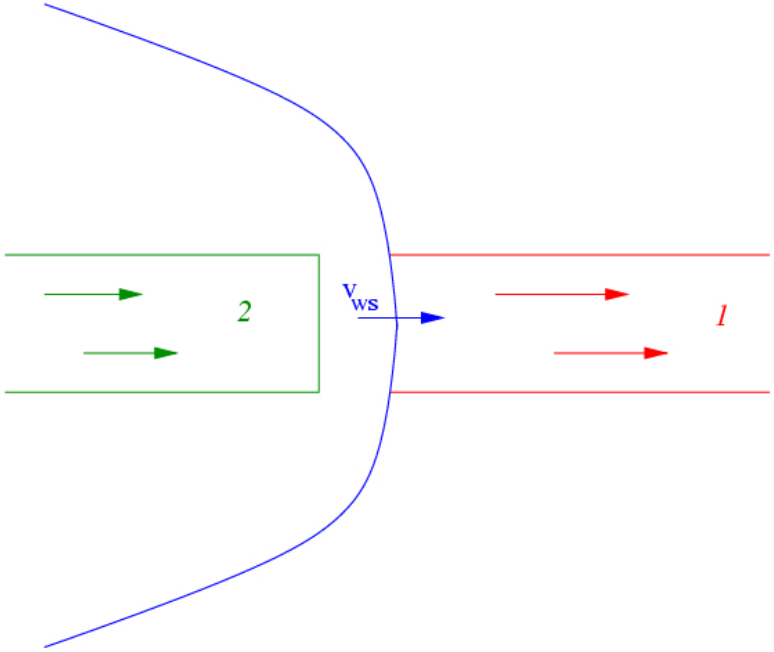
=> INITIAL DISCONTINUITY

- Assume that:

$$u_1 := u_1(\tau_1), \qquad u_2 := u_2(\tau_2) = u_1 + \alpha\Delta t. \tag{6}$$

  ⇒ if $\alpha > 0$ then a working surface is formed.

- Finally, consider *no mass loss* of a *thin* working surface.
- The position of the centre of mass is given by



  - Upstream flow: $x_{\mathrm{ws}} = u_1(t - \tau_2)$
  - Downstream flow: $x_{\mathrm{ws}} = u_2(t - \tau_2)$

7 Dynamics of the working surface

- ☆ Velocity of the working surface = velocity of the centre of mass, i.e.

$$v_{\rm ws} = \frac{1}{M_\gamma} \int_{\tau_1}^{\tau_2} \gamma\left(u(t)\right) \dot{m}(t) u(t) {\rm d}t, \tag{7}$$

with a “weighted” mass

$$M_\gamma = \int_{\tau_1}^{\tau_2} \gamma\left(u(t)\right) \dot{m}(t) {\rm d}t. \tag{8}$$

- ☆ With all these, the position of the shock wave is given by

$$x_{\rm ws} = (t-\tau_2) v_{ws} + \frac{1}{M_\gamma} \int_{\tau_1}^{\tau_2} \gamma\left(u(t)\right)\, \dot{m}(t)\, u(t)\, (t-\tau_2)\, {\rm d}t. \tag{9}$$

- ☆ These equations are parametrised by τ_2 only.

- Initial energy of the flow is

$$E_0 = \int_{\tau_1}^{\tau_2} \dot{m}(\tau)\, \gamma\left(u(\tau)\right)\, c^2 \mathrm{d}\tau, \tag{10}$$

- Energy inside the working surface is $E_{\mathrm{ws}} = m \gamma_{\mathrm{ws}} c^2$.
- The energy loss is then $E_{\mathrm{r}} = E_0 - E_{\mathrm{ws}}$.
- If all this energy is radiated away, then

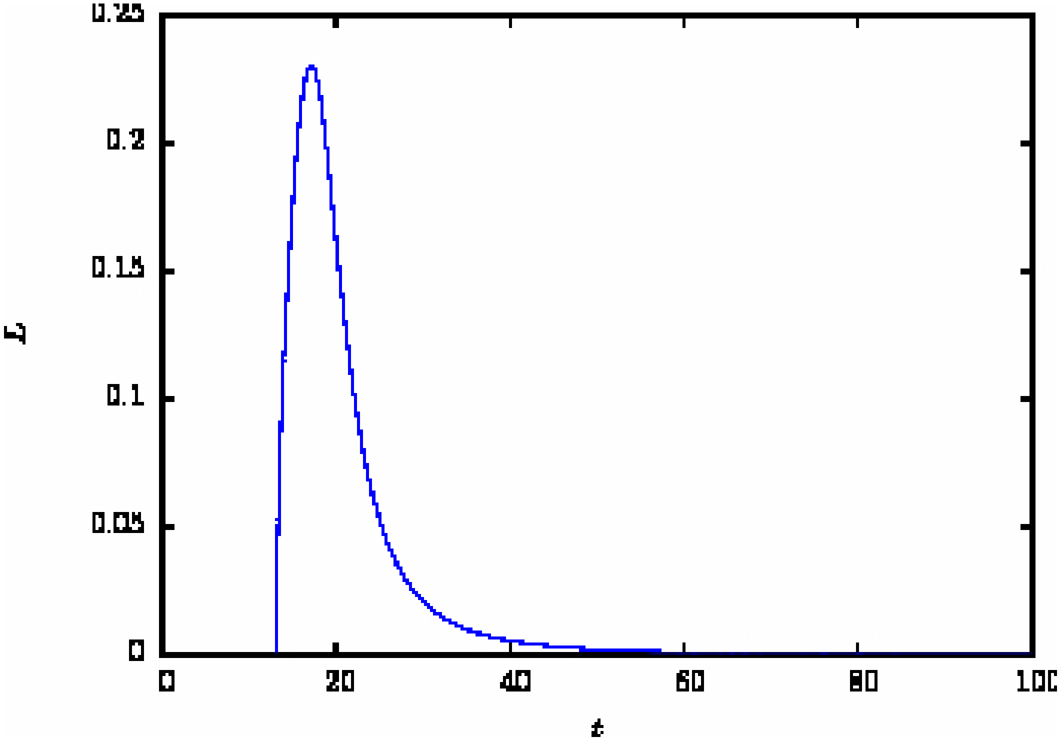
$$\begin{aligned} L &= \frac{\mathrm{d}E_{\mathrm{r}}}{\mathrm{d}t} \\ &= \frac{\dot{m}(\tau_2)}{\mathrm{d}t/\mathrm{d}\tau_2} \left\{ \gamma_{\mathrm{ws}} + \frac{m}{M_\gamma} \gamma_{\mathrm{ws}}^3 \gamma_2 \left(v_{\mathrm{ws}} v(\tau_2) - v_{\mathrm{ws}}^2 \right) - \gamma_2 \right\} - \\ &- \frac{\dot{m}(\tau_1)}{\mathrm{d}t/\mathrm{d}\tau_2} \frac{\mathrm{d}\tau_1}{\mathrm{d}\tau_2} \left\{ \gamma_{\mathrm{ws}} + \frac{m}{M_\gamma} \gamma_{\mathrm{ws}}^3 \gamma_1 \left(v_{\mathrm{ws}} v(\tau_1) - v_{ws}^2 \right) - \gamma_1 \right\}, \\ &= F\left(\dot{m}(\tau_{1,2}), \gamma_{1,2,\mathrm{ws}}, u(\tau_{1,2}), v_{\mathrm{ws}}; \tau_2 \right). \end{aligned} \tag{11}$$

8 A simple example

- $1 = c = \dot{m} = w$, with an oscillating frequency w such that $[w] = 1/t$. This means that

$$[L] = [\dot{m}].v(\tau) = 0.99 - \epsilon^2 \sin\tau, \qquad \epsilon^2 = 0.09$$

- Analytical solution is possible to $O(\gamma^{-1})$ but quite cumbersome!!!
- Semi-analytic solutions are easy (with the aid of the GSL C library).



9 aztekas.org

- ✫ Aztekas code (Mendoza, Olvera, ... 2009)
- ✫ The code is free (in the sense of the Free Software Foundation -GPL license), written on C. It uses maxima, gsl, perl, bash, gnuplot, mencoder & compiled with gcc.
- ✫ At the moment it is capable of solving the 1DRHD using finite differences and artificial viscosity in planar and spherical symmetries.
- ✫ Keep an eye on the site for further updates.

$$\frac{\partial \gamma n}{\partial t} + \frac{1}{r^k}\frac{\partial \gamma n v}{\partial r} = 0, \tag{12}$$

$$\frac{\partial}{\partial t}\left(\frac{e + v^2 p}{1 - v^2}\right) + \frac{1}{r^k}\frac{\partial}{\partial r}\left[r^k v \frac{p + e}{1 - v^2}\right] = 0, \tag{13}$$

$$\frac{\partial}{\partial t}\left(v \frac{p + e}{1 - v^2}\right) + \frac{1}{r^k}\frac{\partial}{\partial r}\left[r^k v^2 \frac{p + e}{1 - v^2}\right] + \frac{\partial p}{\partial r} = 0. \tag{14}$$

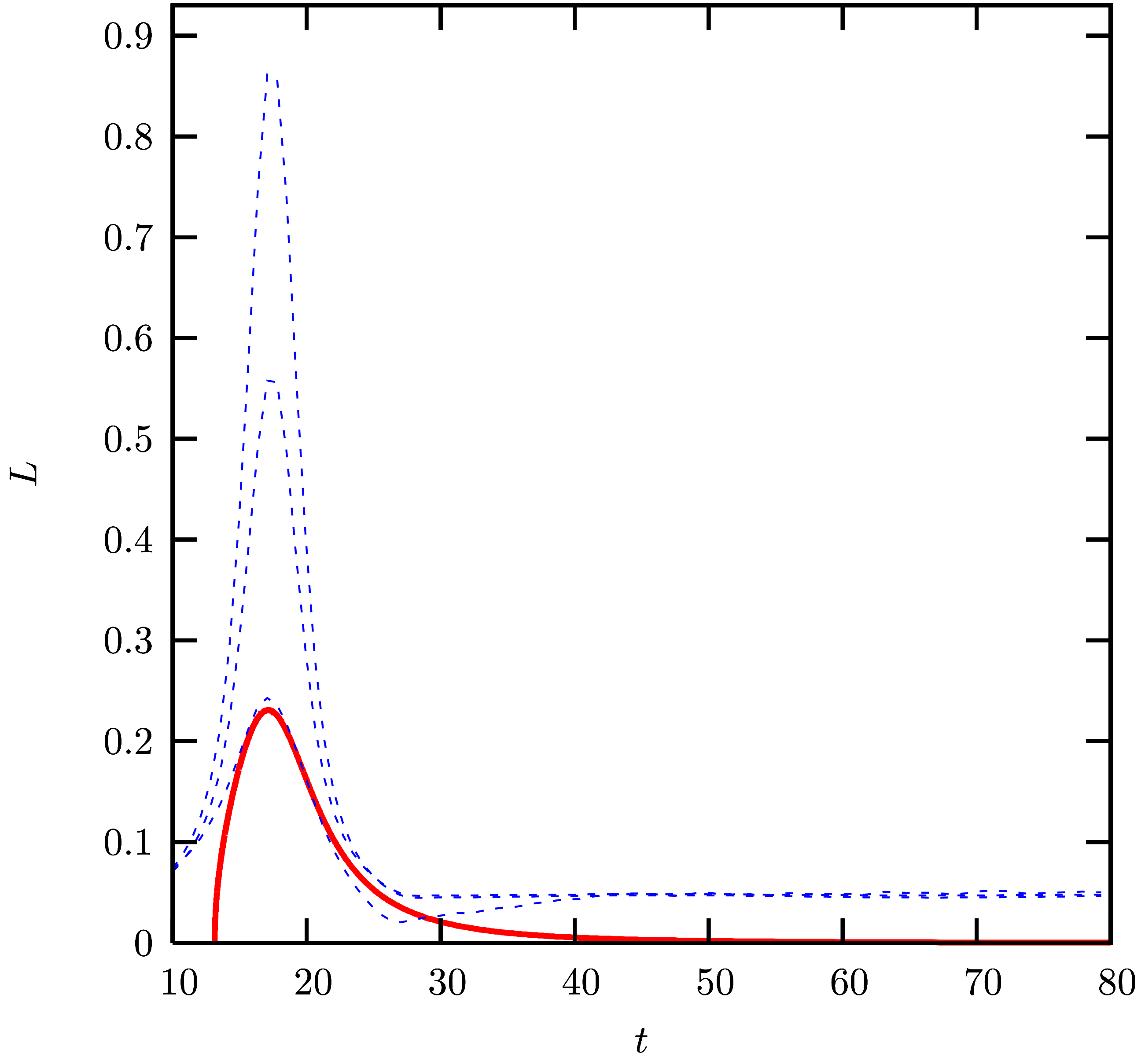
10 Numerical comparison

- $\dot{n} = 1$ & $v(t)$ as analytic case.
- Only a flash of luminosity, $\Rightarrow v(t > 2\pi, x = 0) = 0$.
- Initial conditions: $v(t = 0, x) = 0.9$, $p(t = 0, x) = 0.001$, $n = \dot{n}/\gamma(t = 0, x)v(t = 0, x)$
- We also checked two more cases by using: $p \to \zeta p$ with $z = 0.1,\ 0.2$.
- Initial energy given by:

$$E_0 = \int_0^t \left[1 + \left(3 + v^2\right)\frac{1}{n}\frac{\mathrm{d}p}{\mathrm{d}t} - \left(3 + v^2\right)\frac{p}{n^2}\right]\gamma \mathrm{d}t, \tag{15}$$

- Once the shockwaves are captured, the energy between them is given by:

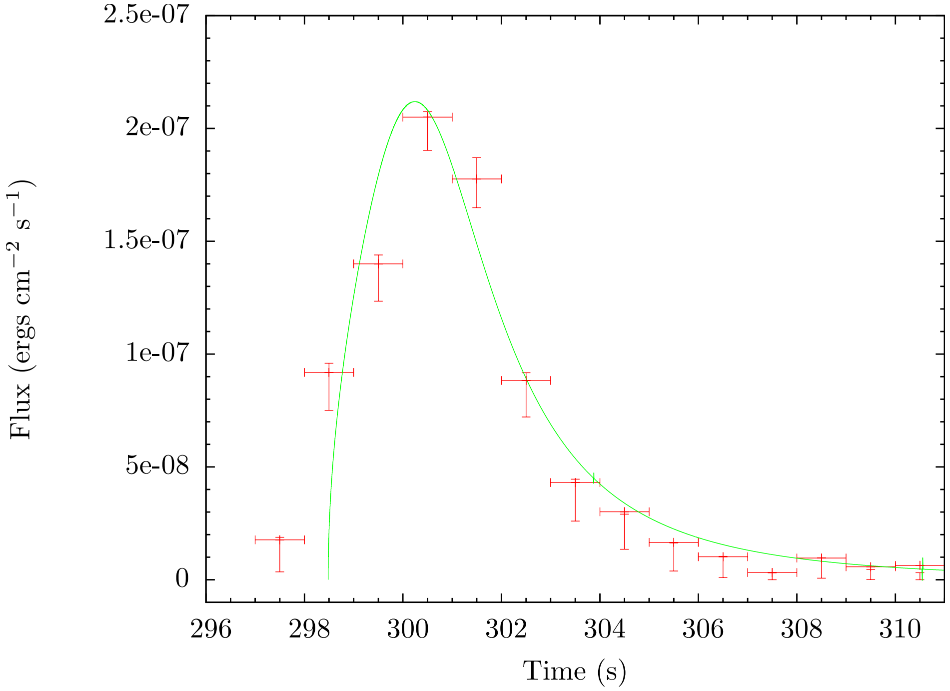
$$E_{\mathrm{ws}} = \sum n\gamma\Delta x + \sum p\gamma\left(3 + v^2\right)\Delta x. \tag{16}$$

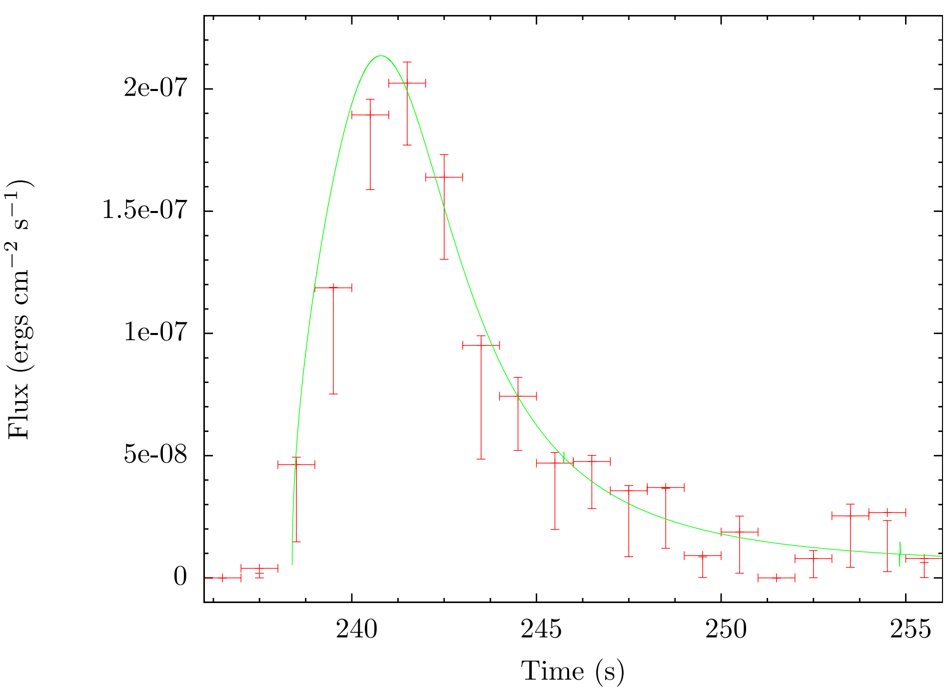


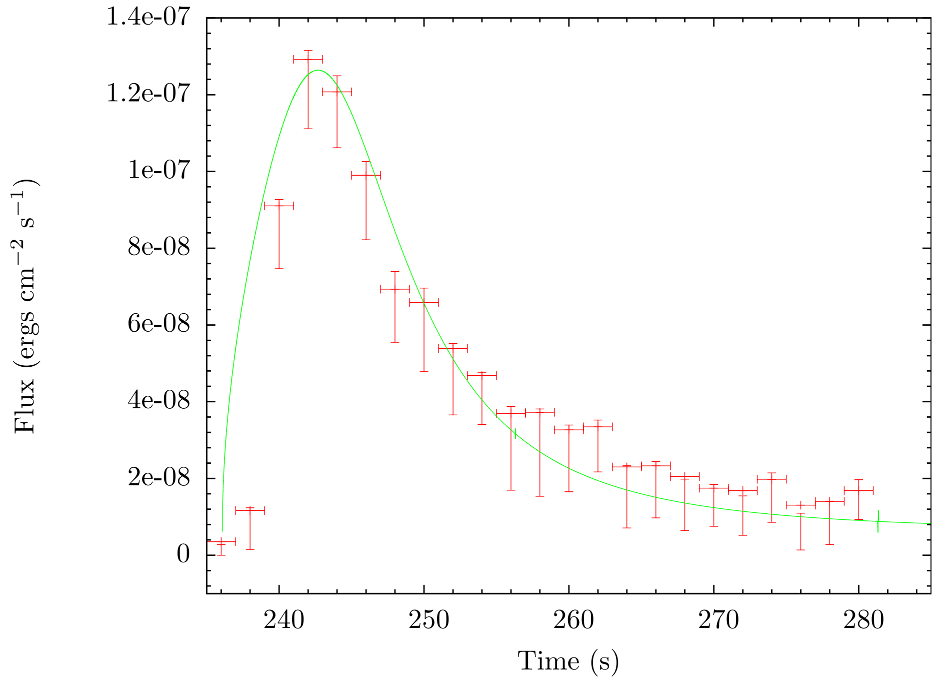
L
t
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
10
20
30
40
50
60
70
80

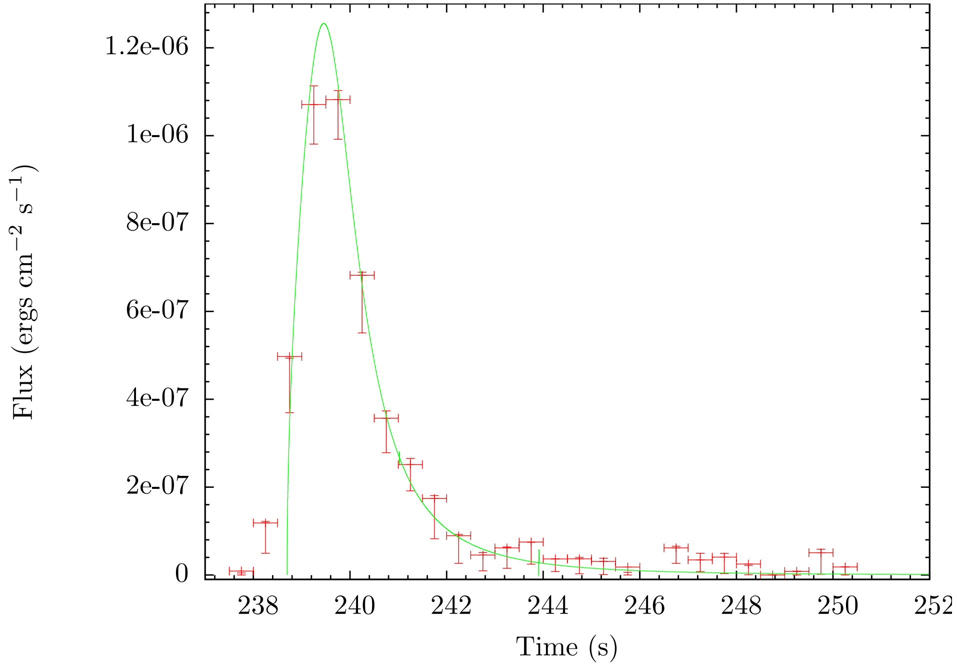
11 Light curves (Mendoza et al. 2009)

- We used 5 long GRBs with simple light curves.
- Fluxes and times are observed.
- Fits are obtained using a simple sinusoidal velocity.
- Linear adjustment to the observed data implies knowing $\dot{m}$ and w for each GRB.



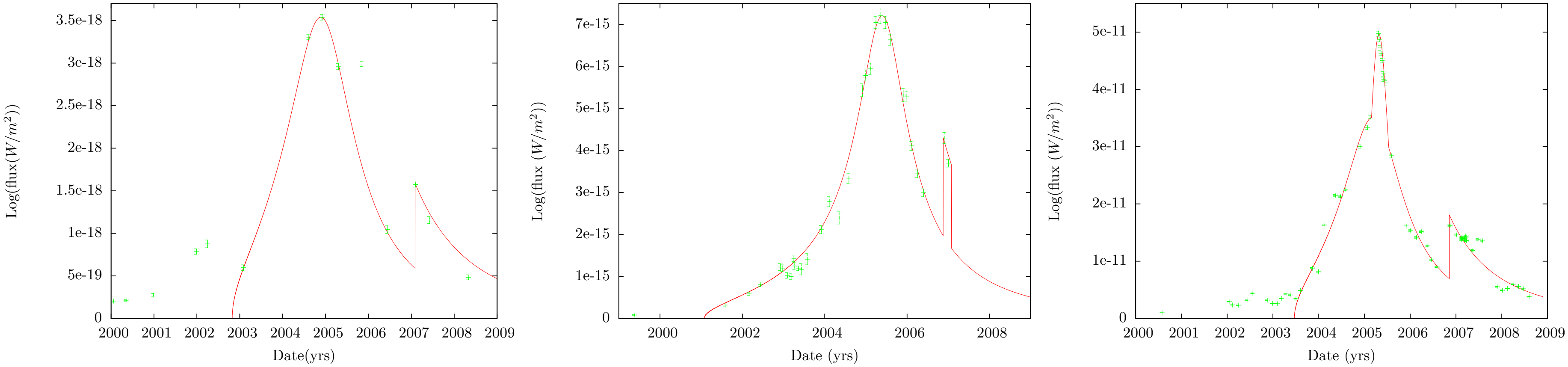



1.4e-07
1.2e-07
1e-07
8e-08
6e-08
4e-08
2e-08
0
Flux (ergs cm^{-2} s^{-1})
240
250
260
270
280
Time (s)

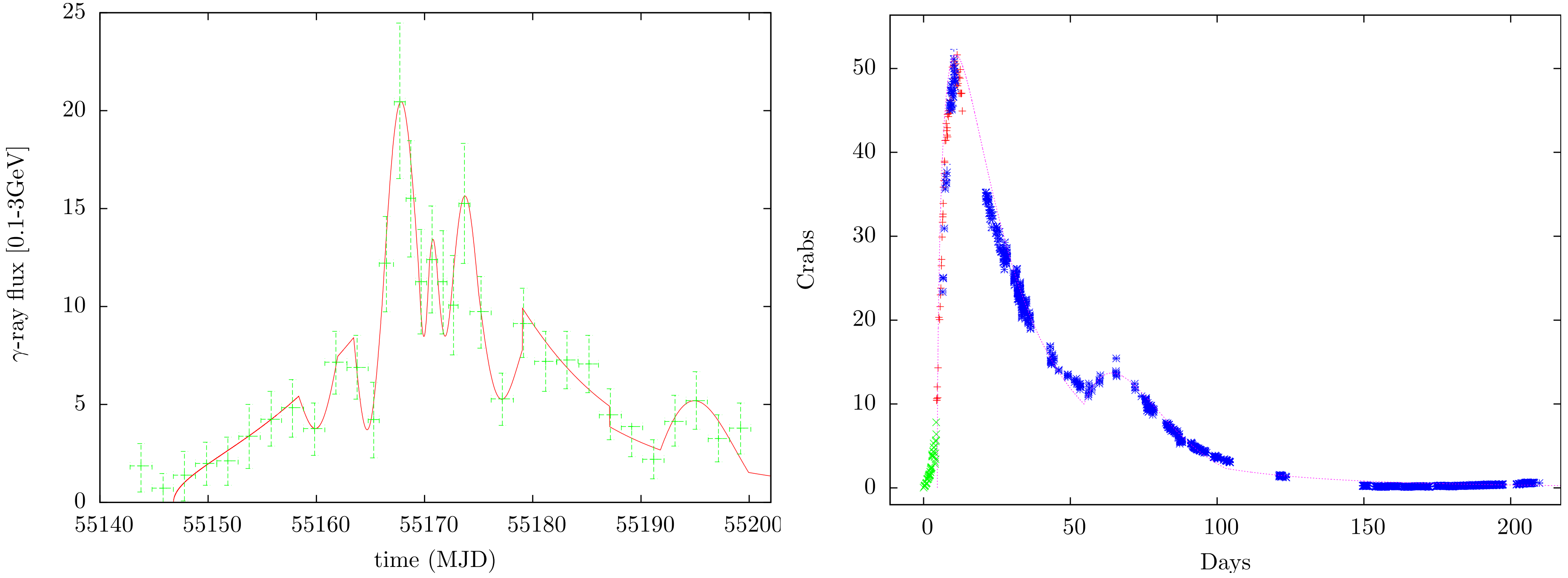

1.2e-06
1e-06
8e-07
6e-07
4e-07
2e-07
0
Flux (ergs cm^{-2} s^{-1})
238
240
242
244
246
248
250
252
Time (s)

12 M87-HST1 knot

- Shows apparent superluminal motion.
- Light curves have been produced recently in radio, UV & X (Harris 2003, 2006, 2009, Madrid 2009, Chang et al. 2010, Chung et al. 2010, Perlman 2003, Wates 2005).
- Fit observations of HST1 with our model (Coronado & Mendoza 2011).



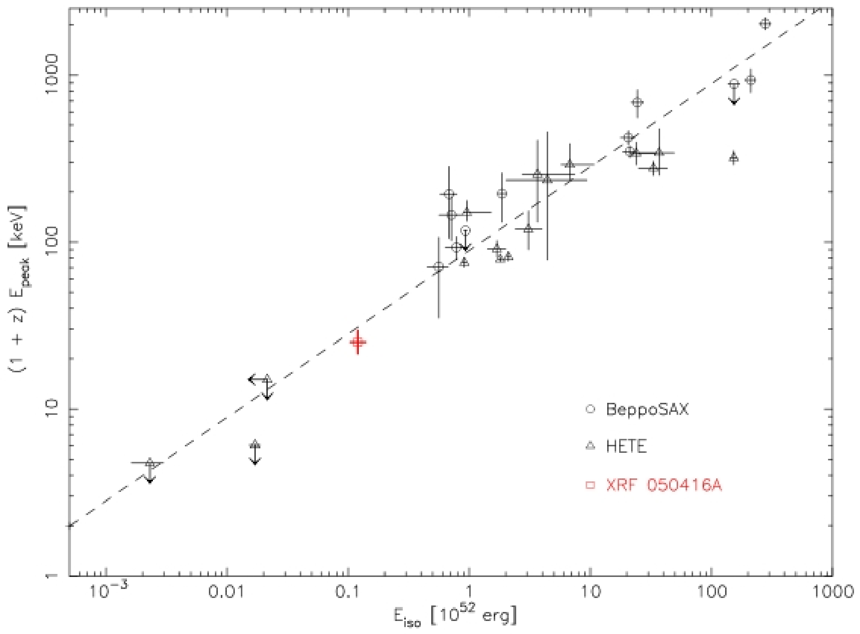
HST1 in radio, UV and X



Fits to 3C454.3 (γ-rays) and A06200 (radio)

13 Observational cosmology with long GRBs?

- SNIa are standard distance indicators for $z \lesssim 1.3$.
- Can long γ-ray bursts be used as distance indicators at $z > 1.3$ and build a Hubble diagram, say at $z > 1.3$?
- Difficult task since GRBs are not "standard candles" at all!
- Ghirlanda, Ghisellini, Firmani . . . 2008, 2009, 2010 have used empirical relations which agree with the concordance model at $z \lesssim 4.5$.



- ☆ Note that GRBs and SNIa should not be considered to be competing standard candles but as complementary cosmological probes in two distinct redshift domains (Firmani et al 2007).
- ☆ All empirical relations are just EMPIRICAL. A model is needed.
- ☆ Using our model we are probing the Universe at $z > 1.3$.

14 Conclusions

- ☆ Shock waves appear on astrophysical flows since many are supersonic. High relativistic accretion/ejection flows appear since the dynamics of flows occur about a high relativistic central objects (e.g. black hole).
- ☆ We have built a simple to use model to understand the light curves of relativistic working surfaces in jets associated to long GRBs, quasars and micro-qsrs.
- ☆ Possible to do cosmology with light curves (stay tunned).

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