Extendiendo la gravitación a nivel relativista

Sergio Mendoza <sergio@mendozza.org>
http://www.mendozza.org/sergio
Instituto de Astronomía
Universidad Nacional Autónoma
de México (UNAM)

Plática disponible en
http://www.mendozza.org/sergio/talks

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Collaborators

- T Bernal JC Hidalgo, LA Torres and DA Carranza (IA-UNAM)
- S Capozziello (Univ. di Napoli))

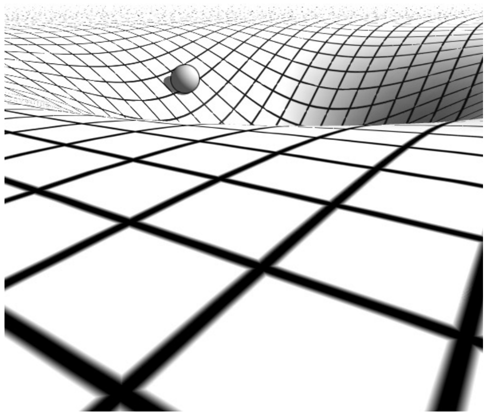
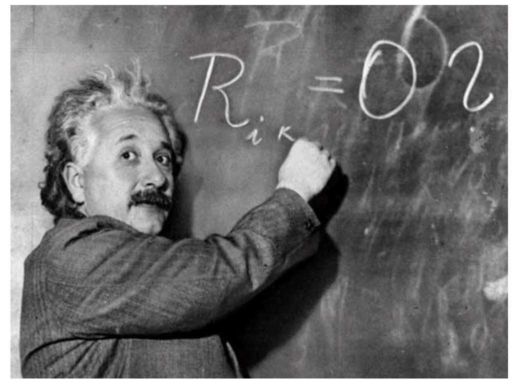
Dimensionality

- Extended relativistic gravity means that the following parameters have to be introduced into the theory:
  - M. Mass of the object producing gravitational field
  - G. Newton's gravitational constant
  - c. Speed of light
  - a_0. Milgrom's acceleration constant.

  $\Longrightarrow$ possible to build two characteristic lengths:

$$\text{Mass length scale } l_M := \left(\frac{GM}{a_0}\right)^{1/2},$$

$$\text{Gravitational radius } r_{\text{g}} := \frac{GM}{c^2}.$$


$$\delta\left(S_{\mathrm{H}}+S_{\mathrm{m}}\right)=0, \tag{1}$$

$$S_{\mathrm{H}}=-\frac{16\pi G}{c^{3}}\int R\sqrt{-g}\,\mathrm{d}^{4}x,\qquad S_{\mathrm{m}}=-\frac{1}{2c}\int \mathcal{L}_{\mathrm{m}}\sqrt{-g}\,\mathrm{d}^{4}x=0. \tag{2}$$

Null variations of the whole action give Einstein field equations:

$$G_{\mu\nu}=R_{\mu\nu}+\frac{1}{2}Rg_{\mu\nu}=\frac{8\pi G}{c^{4}}T_{\mu\nu}, \tag{3}$$

where $\delta S_{\mathrm{m}}=-\left(1/2c\right)T_{\alpha\beta}\,\delta g^{\alpha\beta}$.

☆ Generally speaking, Hilbert action may be written as:

$$S_{\mathrm{H}} = -\frac{c^3}{16\pi G}\int f(R)\sqrt{-g}\,\mathrm{d}^4x, \tag{4}$$

1 Action and field equations

☆ Hilbert's action for the gravitational field:

$$S_{\mathrm{H}} = -\frac{c^3}{16\pi G L_M^2}\int f(\chi)\sqrt{-g}\,\mathrm{d}^4x, \tag{5}$$

where:

$$\chi := L_M^2 R, \tag{6}$$

and L_M is a "characteristic length" of the theory.

☆ Matter action has its usual form:

$$S_{\mathrm{m}} = -\frac{1}{2c}\int \mathcal{L}_{\mathrm{m}}\sqrt{-g}\,\mathrm{d}^4x, \tag{7}$$

☆ Null variations of complete action (i.e. $\delta\left(S_{\mathrm{H}}+S_{\mathrm{m}}\right)=0$) give field equations:

$$f'(\chi)\,\chi_{\mu\nu}-\frac{1}{2}f(\chi)g_{\mu\nu}-L_M^2\left(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Delta\right)f'(\chi)=\frac{8\pi G L_M^2}{c^4}T_{\mu\nu}, \tag{8}$$

with $\Delta := \nabla^\alpha\nabla_\alpha$ and . In what follows a $(+,-,-,-)$ signature will be used.

☆ Trace of field equations:

$$f'(\chi)\,\chi-2f(\chi)+3L_M^2\,\Delta f'(\chi)=\frac{8\pi G L_M^2}{c^4}T. \tag{9}$$

Note: as long as L_M does not depend on the mass and/or energy (i.e. not a function of $T_{\mu\nu}$) the standard "mass tells space how to curve" holds.

2 MONDian solution

☆ Assume $f(\chi) = \chi^b$ and mass is a point mass source M only. To order of magnitude, the trace equation is given by:

$$\chi^b (b-2) + 3bL_M^2 \frac{\chi^{(b-1)}}{r^2} \approx \frac{8\pi GML_M^2}{c^2 r^3}. \tag{10}$$

☆ On the LHS of (10), (2nd term) $\gg$ (1st term) when:

$$Rr^2 \ll \frac{3b}{2-b}. \tag{11}$$

☆ Using the fact that $R \approx \kappa = R_{\mathrm{c}}^{-2}$ then, condition (11) is:

$$R_c \gg r. \tag{12}$$

which corresponds to the relativistic MONDian regime.

☆ Using (12) and the fact that

$$R \approx -\frac{2}{c^2} \nabla^2 \phi \approx -\frac{2\phi}{c^2 r^2} \approx \frac{2a}{c^2 r}, \qquad \Longrightarrow \tag{13}$$

$$
\begin{aligned}
a &\approx \frac{c^2 r}{2L_M^2}\left(\frac{8\pi GM}{3bc^2 r}\right)^{1/(b-1)} \\
&\approx c^{(2b-4)/(b-1)} r^{(b-2)/(b-1)} L_M^{-2} \left(GM\right)^{1/(b-1)} .
\end{aligned} \tag{14}
$$

☆ Choosing $b = 3/2$ gives:

$$
a \approx \frac{\left(GM\right)^2}{c^2 L_M^2\, r}. \tag{15}
$$

☆ The characteristic length L_M has to be formed from the two characteristic lengths of the theory:

$$
L_M \propto r_{\mathrm{g}}^{\alpha} l_M^{\beta}, \qquad \text{with } \alpha + \beta = 1. \tag{16}
$$

☆ ¡The weakest limit of a relativistic theory should not include the speed of light c on the acceleration!

$$
\Longrightarrow \qquad L_M \propto c^{-1}, \qquad \Longrightarrow \qquad \boxed{L_M \propto \left(r_{\mathrm{g}}\, l_M\right)^{1/2}.} \tag{17}
$$

☆ Final result is a MONDIAN acceleration on its standard form:

$$a \approx \frac{(a_0 GM)^{1/2}}{r}, \tag{18}$$

☆ From this equation and the fact that $R \approx 2a/c^2 r$ then

$$R \approx \frac{r_{\rm g}}{l_M} \frac{1}{r^2}, \tag{19}$$

and so, the inequality $R_c \gg r$ is equivalent to

$$l_M \gg r_{\rm g}, \tag{20}$$

which is the expected MONDian limit on the relativistic regime according to X Hernandez's talk (see also Mendoza et al. 2010).

Note. This exact solution can also be formally obtained at the lowest perturbation analysis of the theory and the theory is coherent with Noether symmetries (see arXiv:1108.5588).

Hilbert's action depends on L_M which itself depends on the mass of the system. This violates Hilbert's postulate on the gravitational field.

"..one should not be surprised if some of the commonly accepted notions, even at the fundamental level of the action require generalisations and rethinking."

$$S_{\rm H} = -\frac{c^3}{16\pi G}\int \frac{f(\chi)}{L_M^2}\sqrt{-g}\,\mathrm{d}^4x, \qquad \chi := L_M^2 R, \tag{21}$$

$$L_M = \frac{2\sqrt{2}}{9}\left(r_{\rm g}\, l_M\right)^{1/2}, \qquad r_{\rm g} := \frac{GM}{c^2}, \qquad l_M := \left(\frac{GM}{a_0}\right)^{1/2}. \tag{22}$$

$$f(\chi) = \begin{cases} \chi & \Longrightarrow \quad \text{general relativity} \\ \chi^{3/2} & \Longrightarrow \quad \text{extended relativistic MOND} \end{cases}$$

$\Longrightarrow$ Field's action depends on the mass/energy.

3 $f(R,T)$ gravity connection

☆ Harko et al. (2011) have developed an $F(R,T)$ gravity theory, with $T := T^{\alpha}_{\alpha}$ and so:

$$F(R,T) := f(\chi)/L_M^2, \qquad \text{and “mass-energy”} \qquad M := \frac{1}{c^2}\int T\,\mathrm{d}^3x \tag{23}$$

☆ Field equations for a perfect fluid in this case are:

$$\begin{aligned} f_R R_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} + L_M^2 \left(g_{\mu\nu}\Delta - \nabla_\mu\nabla_\nu\right)\left(f_R/L_M\right) \\ = \frac{8\pi G L_M^2}{c^4} T_{\mu\nu} + L_M^2 \left(f/L_M^2\right)_T \left(T_{\mu\nu} + p g_{\mu\nu}\right). \end{aligned} \tag{24}$$

- ☆ E.g. dust model in cosmology (FLRW). Friedman's equation evaluated today is (see paper -arXiv this week)

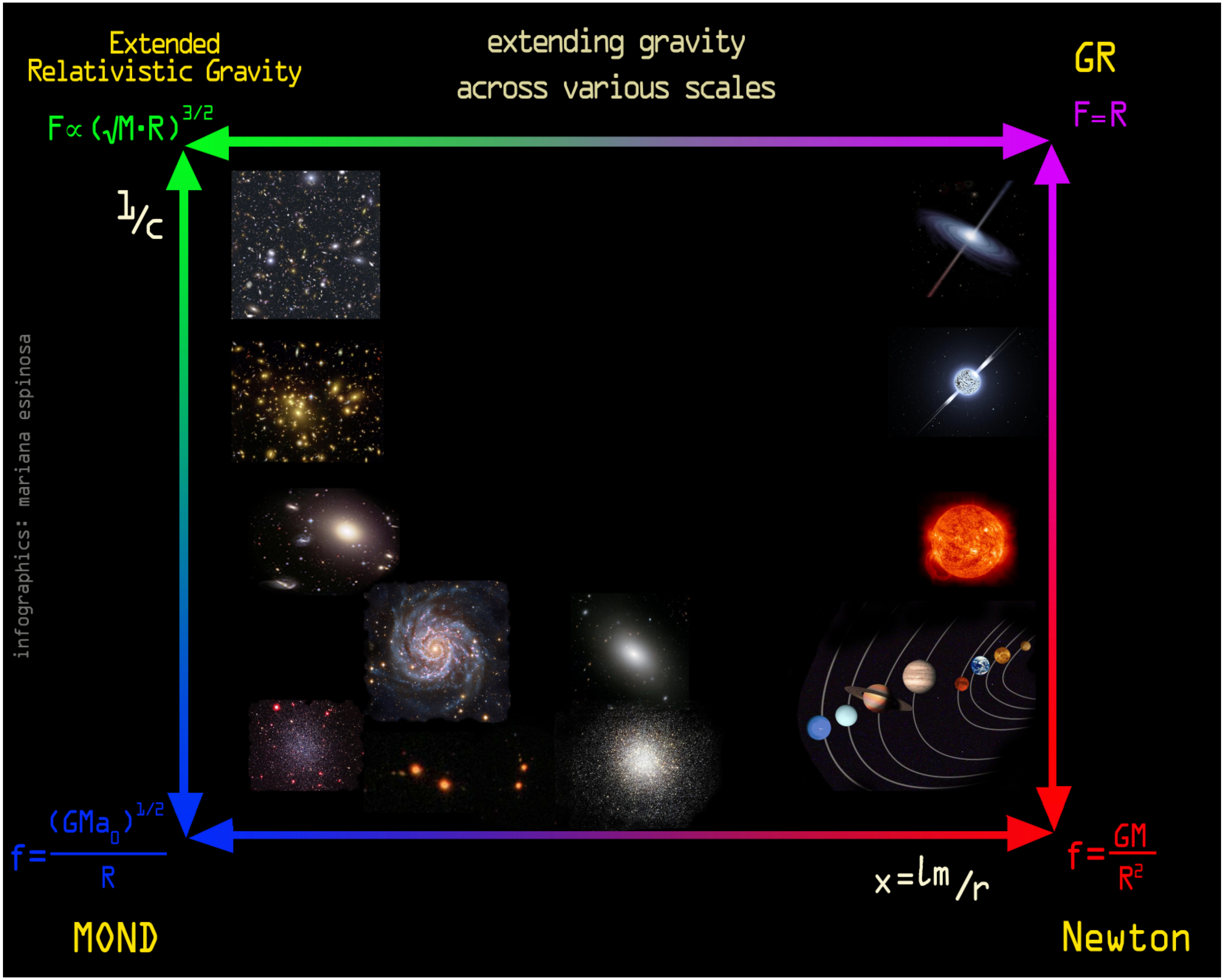
$$\frac{a_0}{cH_0} = \mathcal{Z}\left(b, \Omega_0, j_0\right) \approx 1 \tag{25}$$

for $f(\chi) = \chi^b$.

- ☆ As a final remark, note that the function

$$f(\chi) = \chi^{3/2} \, \frac{1 \pm \chi^{p+1}}{1 \pm \chi^{3/2+p}} \longrightarrow \begin{cases} \chi^{3/2} & \text{when } \chi \ll 1 \quad \text{(Ext. Rel. MOND)} \\ \chi & \text{when } \chi \gg 1 \quad \text{(Gen. Rel.)} \end{cases} \tag{26}$$

for $p \geq -1$.


Extended Relativistic Gravity
extending gravity across various scales
GR
$F \propto (\sqrt{M \cdot R})^{3/2}$
F=R
1/c
infographics: mariana espinosa
$f = \frac{(GMa_0)^{1/2}}{R}$
$x = lm/r$
$f = \frac{GM}{R^2}$
MOND
Newton